Application Note

STAN Tool

Rollet Factor versus Pole-Zero Identification

Understanding their differences for
a more efficient use of both

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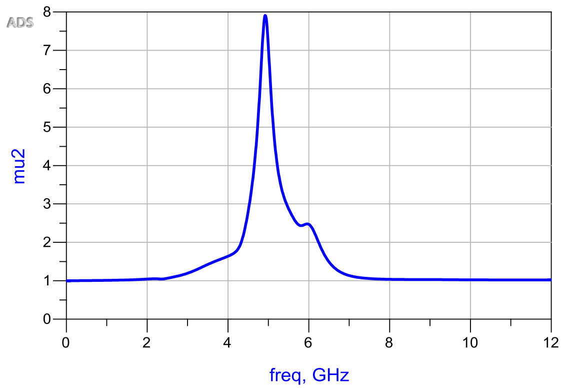
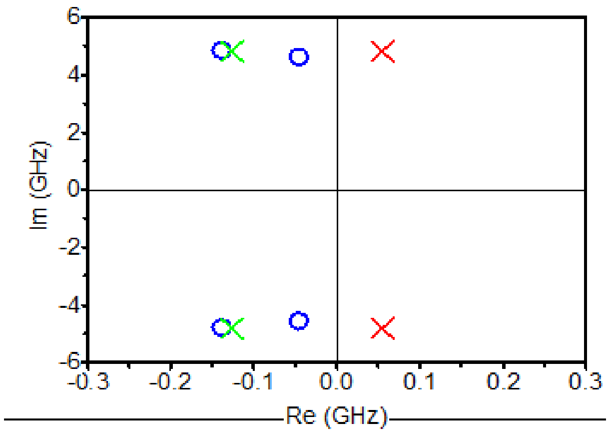
Rollet Factor and Pole-Zero Identification

Rollet factor and **pole-zero identification** are analysis methods used by circuit designers to prevent undesired oscillations in their microwave amplifiers.





In some cases there is a bit of confusion on why or when use a method or the other.

In this application note we will try to explain the main differences between the two methods so as to better understand their scope and limitations.

The goal is helping the designers to use them in the most effective way.

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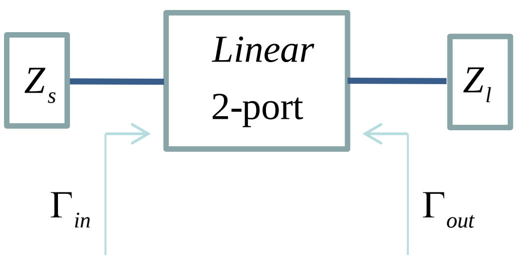
1.1 Rollet criterion / K Factor

Rollet criterion (well-known as **K Factor**) is a useful, widely established stability criterion that tell us whether a linear two-port, which was intrinsically stable, will exhibit at its ports negative resistance for any value of passive source or load impedances (unconditional stability).



$$\left|\Gamma_{in}(\omega)\right| < 1 \quad \left|\Gamma_{out}(\omega)\right| < 1$$

for all passive Z_s and Z_l at all ω

This can be verified with the K and Δ factors

$$K = \frac{1 - \left|S_{11}(\omega)\right|^2 - \left|S_{11}(\omega)\right|^2 + \Delta^2(\omega)}{2\left|S_{21}(\omega)S_{12}(\omega)\right|} > 1$$

$$\left|\Delta(\omega)\right| = \left|S_{11}(\omega)S_{22}(\omega) - S_{21}(\omega)S_{12}(\omega)\right| < 1$$

Or alternately with the μ factor

$$\mu = \frac{1 - \left|S_{11}(\omega)\right|^2}{\left|S_{22}(\omega) - \Delta(\omega)S_{11}^*(\omega)\right| + \left|S_{21}(\omega)S_{12}(\omega)\right|} > 1$$

If the circuit is not unconditionally stable, we can trace the stability circles to determine the region of the smith chart for Z_l (Z_s) that yield $\left|\Gamma_{in}(\omega)\right| > 1 \quad \left(\left|\Gamma_{out}(\omega)\right| > 1\right)$

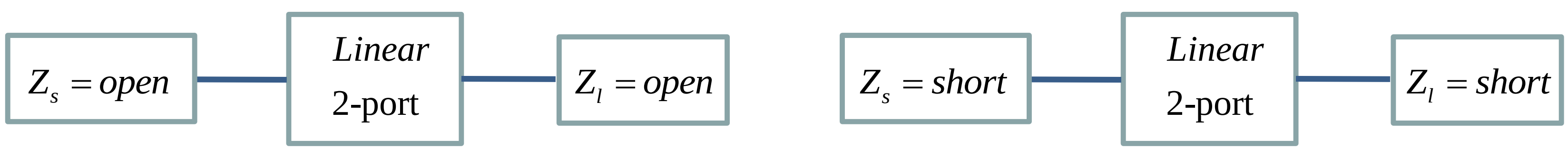
All the considerations made in this app note on the applicability of Rollet criterion also stand for the stability circles, obviously.

1.2 Rollet proviso

Rollet criterion (well-known as **K Factor**) is a useful, widely established stability criterion that tell us whether a linear two-port, which was intrinsically stable, will exhibit at its ports negative resistance for any value of passive source or load impedances (unconditional stability).

This is a key point in the criterion. In order to be applicable, the linear two-port must be stable when both ports are loaded with open and short circuits. In other words, the two-port does not have unstable poles when both ports are loaded with open and short circuits.

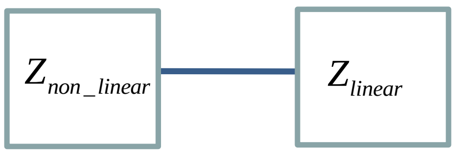
Stable under these two load conditions



This is equivalent to say that the two-port does not have any unstable internal loop whose dynamics is not observable at the ports. **This is the Rollet proviso** that must be previously fulfilled for the Rollet stability criterion to be rigorously applied.

1.3 Rollet criterion is not a stability test

Rollet criterion (well-known as **K Factor**) is a useful, widely established stability criterion that tell us whether a linear two-port, which was intrinsically stable, will exhibit at its ports negative resistance for any value of passive source or load impedances (unconditional stability).



$$\mathrm{Re}\left(Z_{non\_linear}\right) < \mathrm{Re}\left(Z_{linear}\right)$$

$$\mathrm{Im}\left(Z_{non\_linear}\right) = \mathrm{Im}\left(Z_{linear}\right)$$

“**Exhibiting negative resistance** ” does not mean necessarily the startup of an oscillation. Having a negative resistance is only half of the story. For oscillation, the fulfillment of a necessary condition on the imaginary part of the impedances is also required. Thus, the criterion is telling us that, due to this negative resistance, the circuit **might** fulfill the oscillation startup conditions once completed with some actual load and source impedances .

In short, Rollet criterion is not a stability test. It is a stability criterion based on negative resistance analysis. It serves to tell us if the circuit **might oscillate** for some values of passive source and load terminations (potential instability) or if the circuit **will not oscillate** for any value of passive source and load terminations (unconditional stability).

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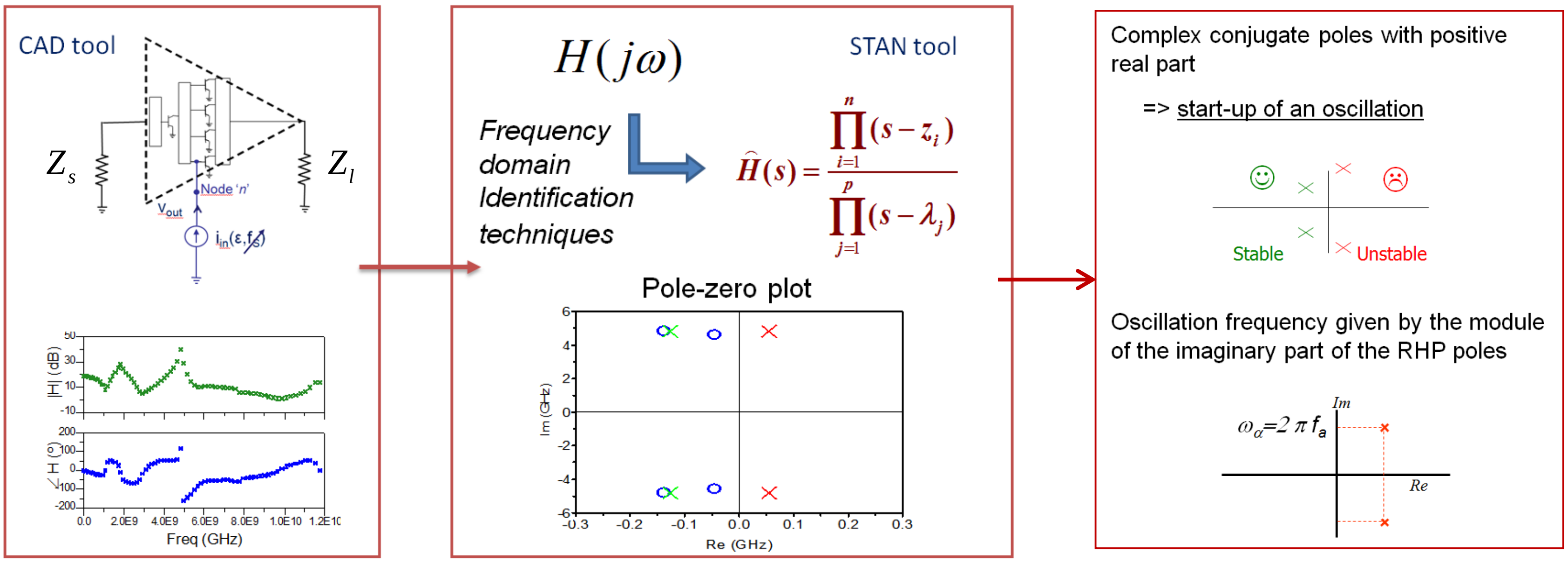
1.4 Pole-zero identification

It is an indirect method to get the eigenvalues of the linear (or linearized system) from the identification of a closed-loop frequency response. It is applied for fixed load and source conditions Zs(ω) and Zl(ω)



Pole-zero identification is a complete stability test. It is able to predict whether a circuit **will or will not** oscillate[1] for fixed source and load terminations $Z_s(\omega)$ and $Z_l(\omega)$.

[1]Actually, as any stability test, it is a local stability analysis . Thus, we should say more accurately that it predicts whether the **startup of an oscillation will or will not** take place.

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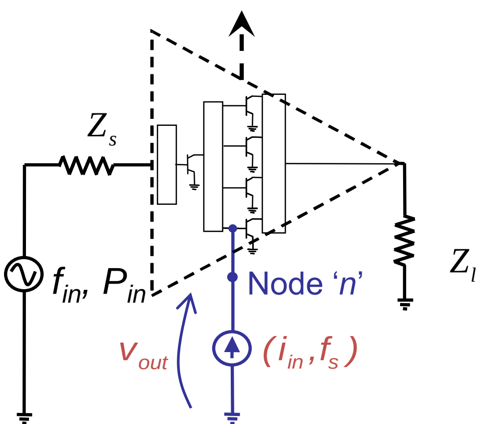
1.5 Stability analysis of large-signal regimes

Rollet Criterion only concerns the stability of a linear two port or a linearized circuit around a *dc* steady state. It cannot be directly applied to analyze the unconditional stability of a an amplifier working on large-signal regime[3].

However **pole-zero identification** can be perfectly applied to the detection of parametric instabilities that are function of the power P_{in} and frequency f_{in} of the input large-signal drive (for fixed load and source conditions $Z_s(\omega)$ and $Z_l(\omega)$).



$$H(j\omega_S) = \frac{v_{out}(\omega_S)}{i_{in}(\omega_S)}$$

Conversion matrix algorithm

$H(j\omega)$

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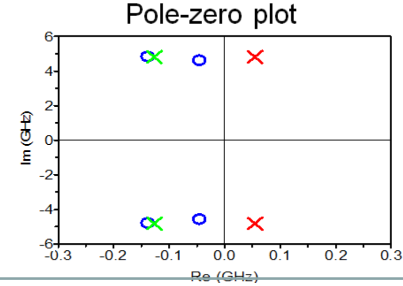
Frequency domain Identification techniques

$$\hat{H}(s) = \frac{\prod_{i=1}^{n}(s - z_i)}{\prod_{j=1}^{p}(s - \lambda_j)}$$

Pole-zero plot



[3]Analyzing the unconditional stability versus source and load terminations of an amplifier in the presence of a large signal at an input frequency f_{in} is an incommensurable multidimensional problem because, for each input power P_{in}, instability at frequency f_s will depend on source and load terminations at K f_{in} and at side bands K $f_{in} \pm f_s$, with K being the number of relevant harmonics. The most rigorous and wise generalization of the Rollet criterion to the stability of large-signal periodic regimes is given in *Suarez et al. "Generalized Stability Criteria for Power Amplifiers Under Mismatch Effects" IEEE Trans. Microw. Theory Tech., vol. 63, pp. 4415-4428, December 2015*. It is applied to practical cases in which the output filter of the amplifier allows the consideration of only two (or three at most) relevant sidebands at the load termination port. Otherwise the analysis becomes impractical.

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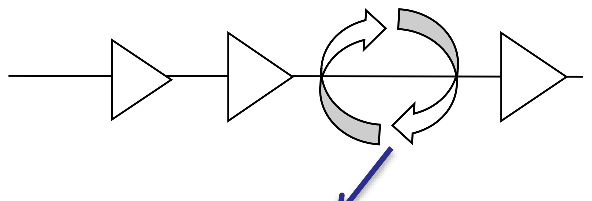
2. Example of an incorrect application of the Rollet criterion

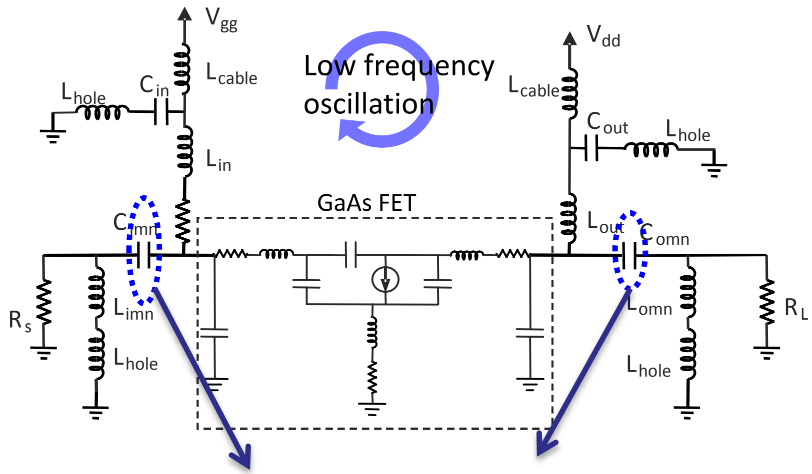
Very often the Rollet criterion (or the stability circles) is applied without verifying the proviso. This can lead to an erroneous stability prediction in those amplifiers with an unstable dynamics isolated (not observable) from the input and output terminals. Two common examples are:

Internal instabilities in **multi-stage power amplifiers**



Unstable loop not observable from the input and output access of the amplifier

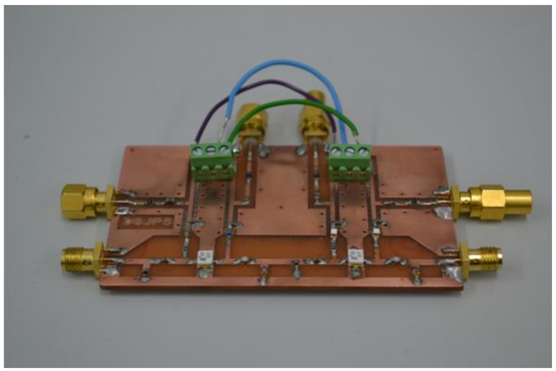
Low frequency instabilities that involve bias elements

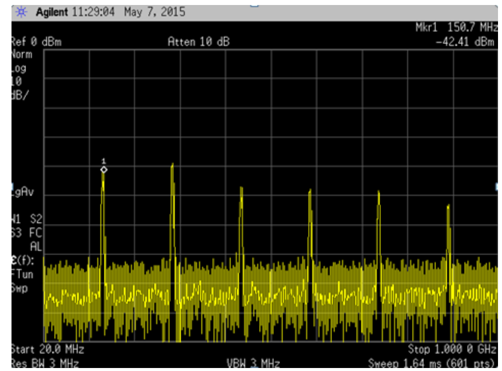


The decoupling RF capacitors can isolate the low frequency dynamics from the RF ports

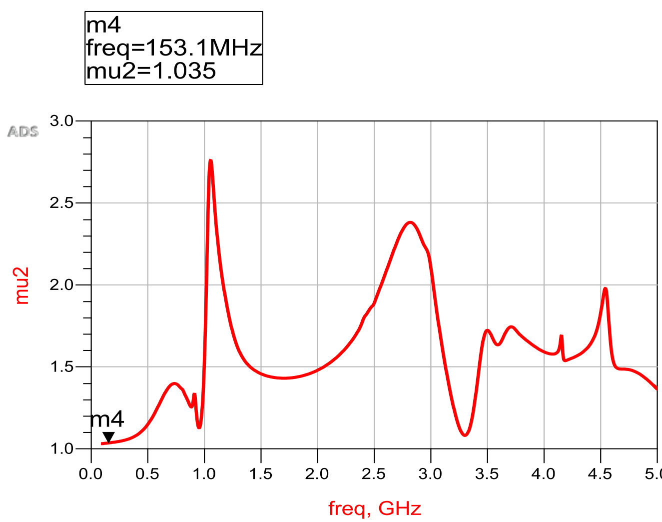
2. Example of an incorrect application of the Rollet criterion

We show here the example of an unstable two-stage band-L FET amplifier. The amplifier presents a spurious oscillation at 150 MHz.





However, the simulated μ factor is > 1 all over the frequency range, and in particular at 150 MHz:



Verifying the proviso is mandatory !!!

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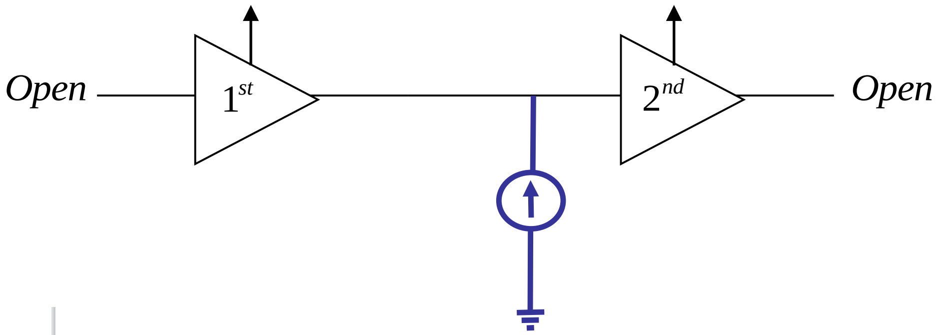
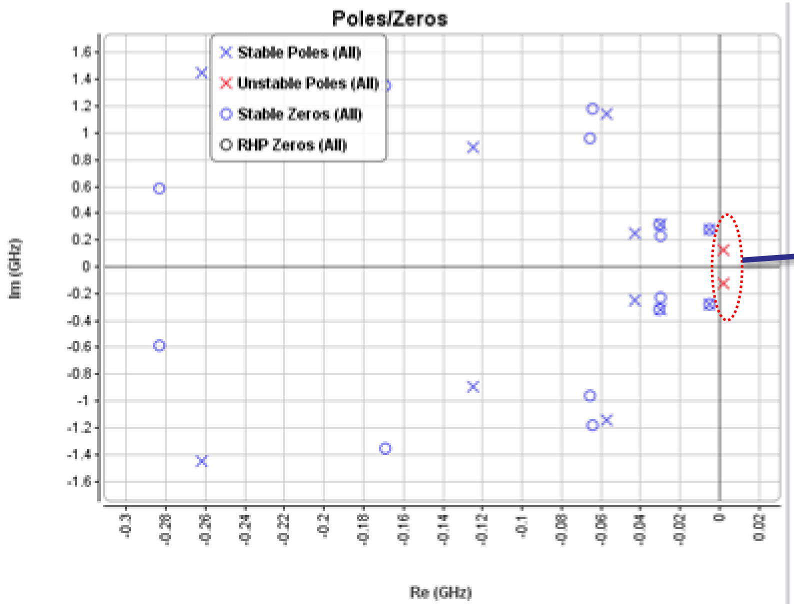
2. Example of an incorrect application of the Rollet criterion

We can use pole-zero identification to verify the proviso in this amplifier

Frequency response is analyzed at an internal node of the two stage amplifier





A pair of unstable complex conjugate poles is obtained at about 150MHz when the amplifier is loaded with open circuits, both input and output. **This confirms the internal instability.**

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3. How to use Rollet criterion appropriately

- First, always use it in its right context → The study of the ***potential instability / unconditional stability*** versus source and load terminations of a linear 2-port circuit.

- Second, always verify the proviso (and specially in the case of multistage amplifiers) → This can be easily done with **pole-zero identification**

Document Outline

1 – Rollet criterion and Pole-Zero identification. Key differences

- 1.1 – Rollet criterion
- 1.2 – Rollet proviso
- 1.3 – Rollet criterion is not a stability test
- 1.4 – Pole-zero identification
- 1.5 – Stability of large-signals regimes

2 – Example of an incorrect application of the Rollet criterion

3 – How to use Rollet criterion appropriately

4 – Summary table

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4. Summary table

Rollet / K factor	Pole-zero
It is not a stability test. It does not conclude on the actual startup of the oscillation because it does not analyze the condition on the imaginary part.	**It is a full stability test**. If Right Half Plane poles are detected the startup of the oscillation is guaranteed.
It is a negative resistance analysis **versus source and load conditions.**	It is a stability test for **fixed terminations $Z_s(\omega)$ and $Z_l(\omega)$**.
It is not directly applicable to circuits operating in **large-signal regime**.	It is applicable to circuits operating in **large-signal regime**.
The Rollet factor alone is not appropriate to predict internal instabilities that are not observable at external ports like can happen in **multistage amplifiers**.	It is applicable to the detection of internal unstable loops in **multistage amplifiers**.
The **proviso** should be always verified, specially in multistage amplifiers.	Can be used to verify the **proviso** on the Rollet criterion